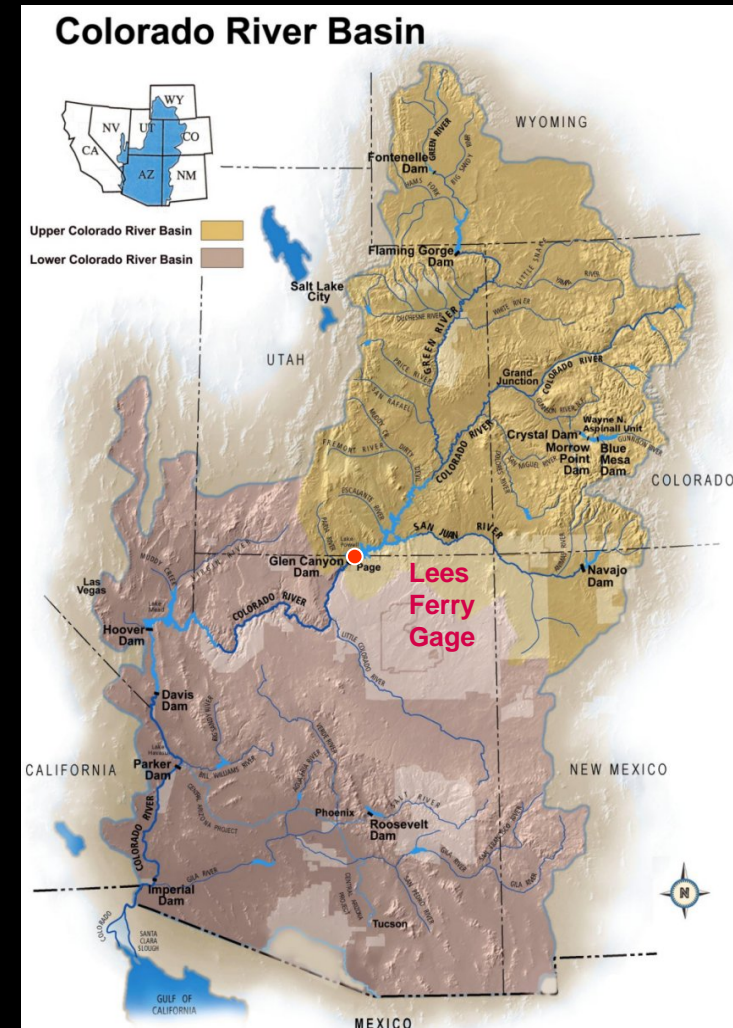


# Non-Parametric Paleo-Reconstruction of Lees Ferry Flows

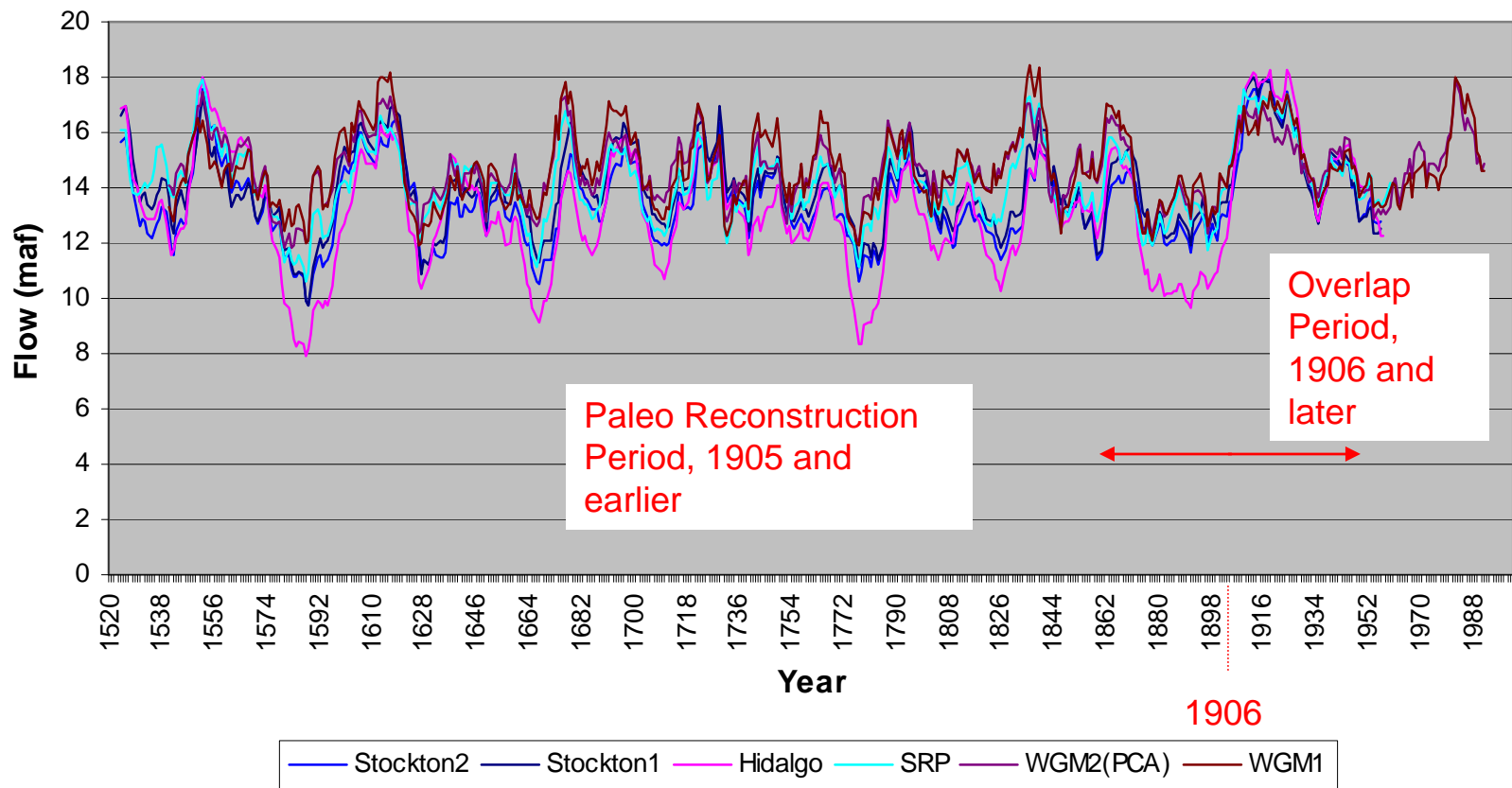
Subhrendu Gangopadhyay, Ph.D., P.E.  
Ben Harding, P.E.

Sponsored by  
U.S. Bureau of Reclamation, and  
CIRES/University of Colorado, Boulder,  
NOAA Western Water Assessment

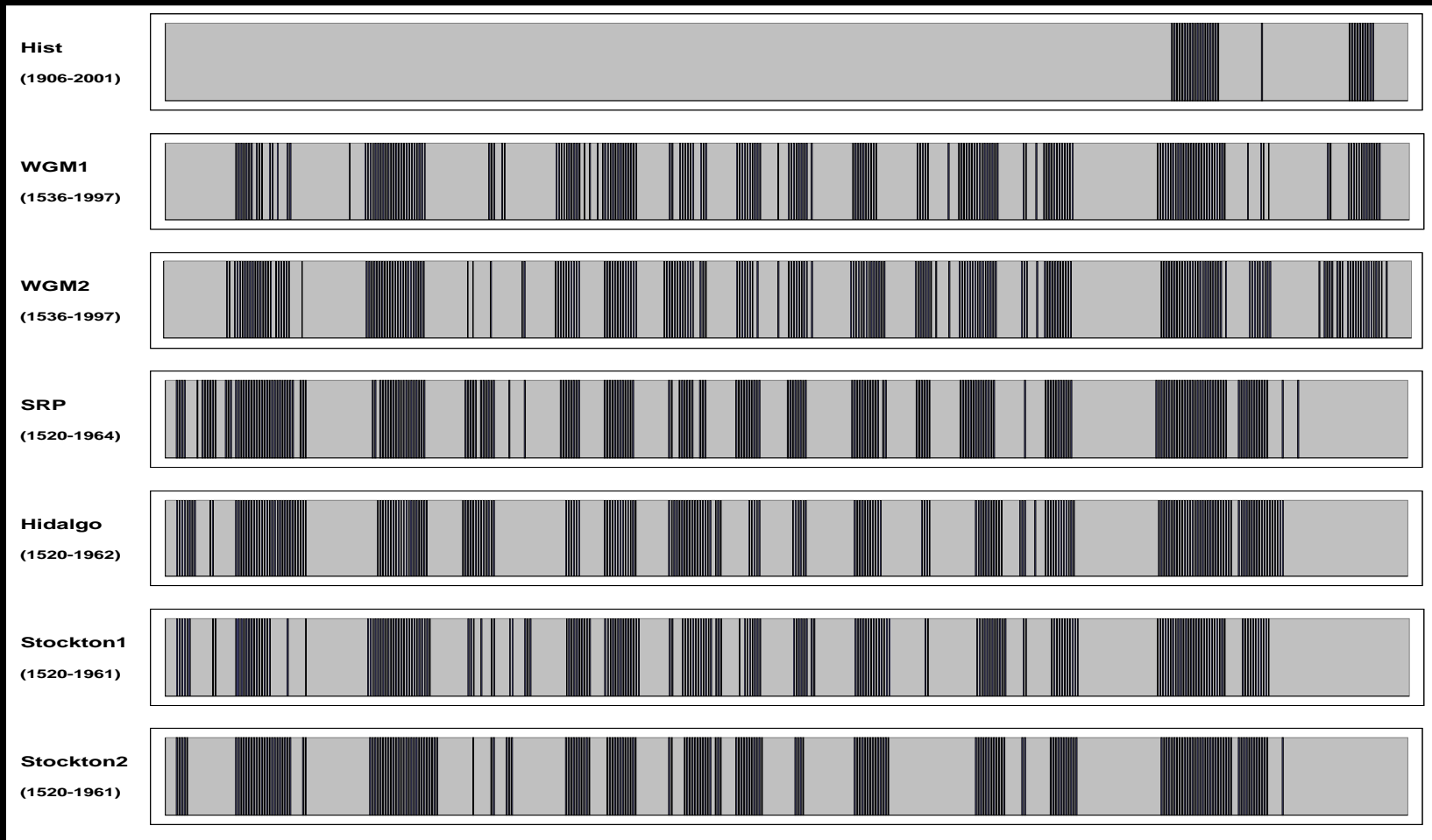


# Existing Lees Ferry Reconstructions

## Reconstructions of Colorado River Flows at Lees Ferry



# Common Signal in Tree-Ring Chronologies – The Bar Codes



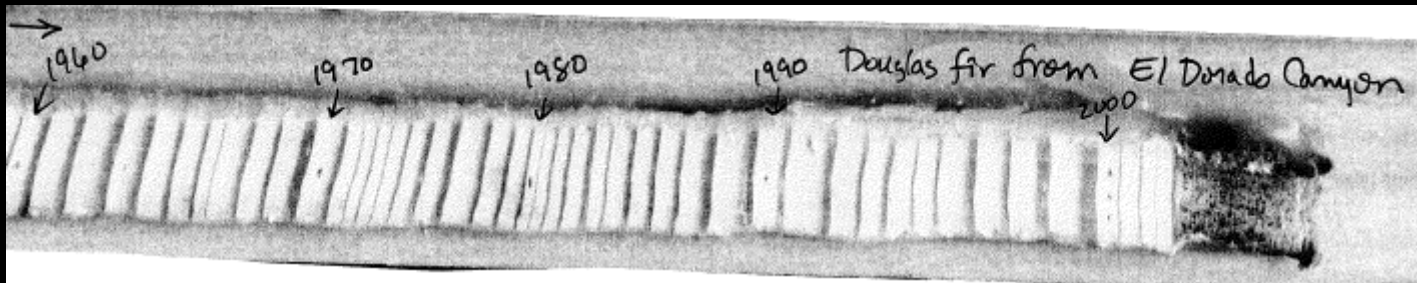
## Objectives

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- Develop a method to reconstruct streamflow using non-parametric techniques.
- Compare NP reconstructions with existing reconstructions of Lees Ferry flows.

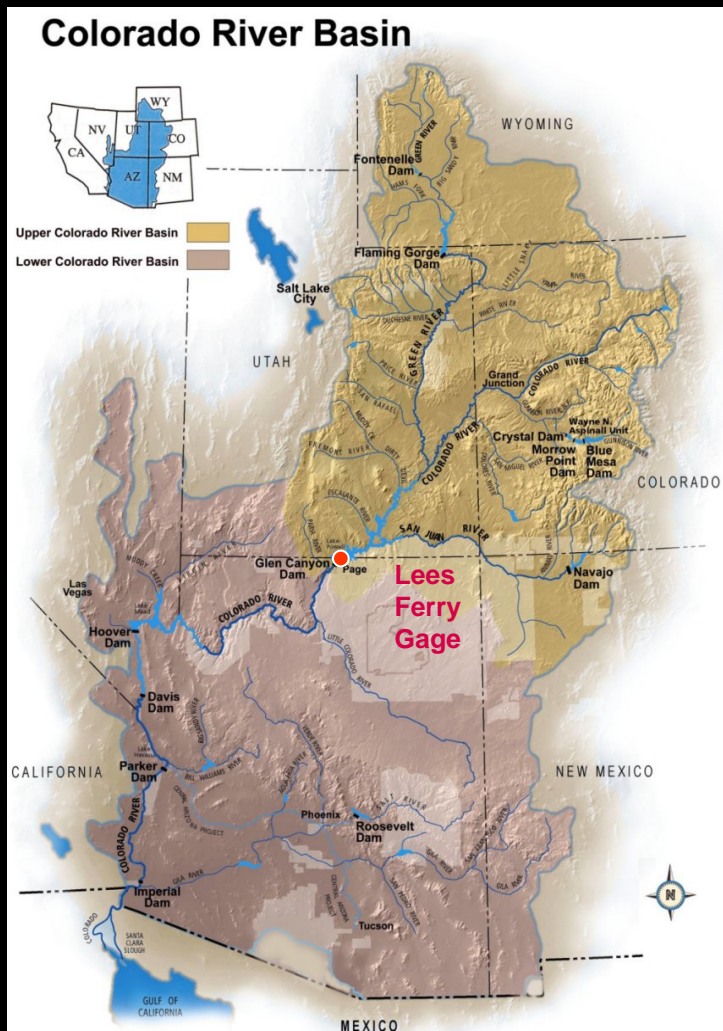
## Datasets - Tree-Ring Site Chronology

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- Growth indices for a site.
- Filtered indices to represent the growth of annual tree-rings from available moisture.
- Annual time series of growth index for a site.
- 51 chronologies for Upper Colorado, ~1400 – 2005  
(Jeff Lucas, CIRES/NOAA).

# Datasets – Naturalized Streamflow at Lees Ferry



- Naturalized streamflow data from, 1906-2005 (*Jim Prairie, USBR*)

# NP Recon Algorithm

<i>Year</i>	Chronology Site 1	Chronology Site 2	...	Chronology Site N
1400	x	x	...	x
1401	x	x	...	x
...	...	...	...	...
2004	x	x	...	x
2005	x	x	...	x

Reconstruction period, 1400 - 1905

= **[X]**, data matrix for the overlap period, 1906 - 2005

## NP Recon Algorithm

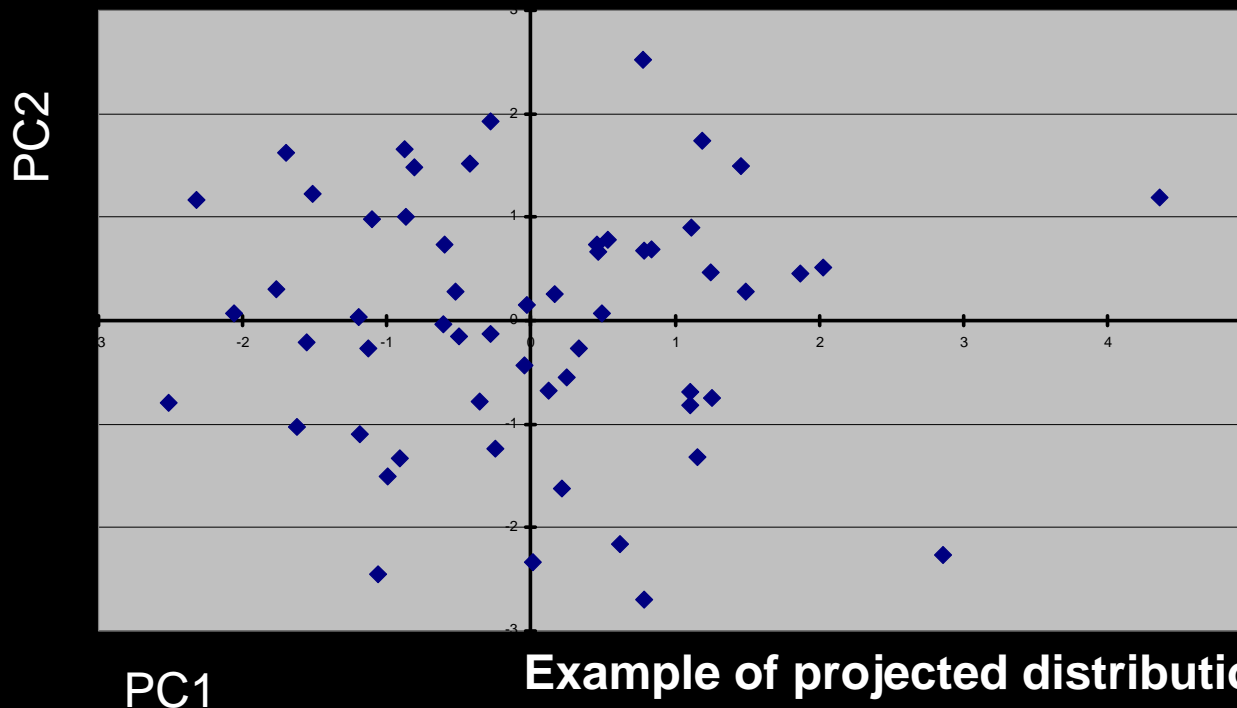
---

- Do eigen decomposition using the correlation matrix of  $X$ .
- Retain the eigen values and eigen vectors, and derive the principal components (time projection of the chronologies).
- For each year of the reconstruction period (1400 – 1905), project the chronologies on to the above eigen vectors and calculate the distances (measure of similarity) with the overlap years.
- Derive weights based on similarity.
- Resample based on weights.



## Similarity Analysis

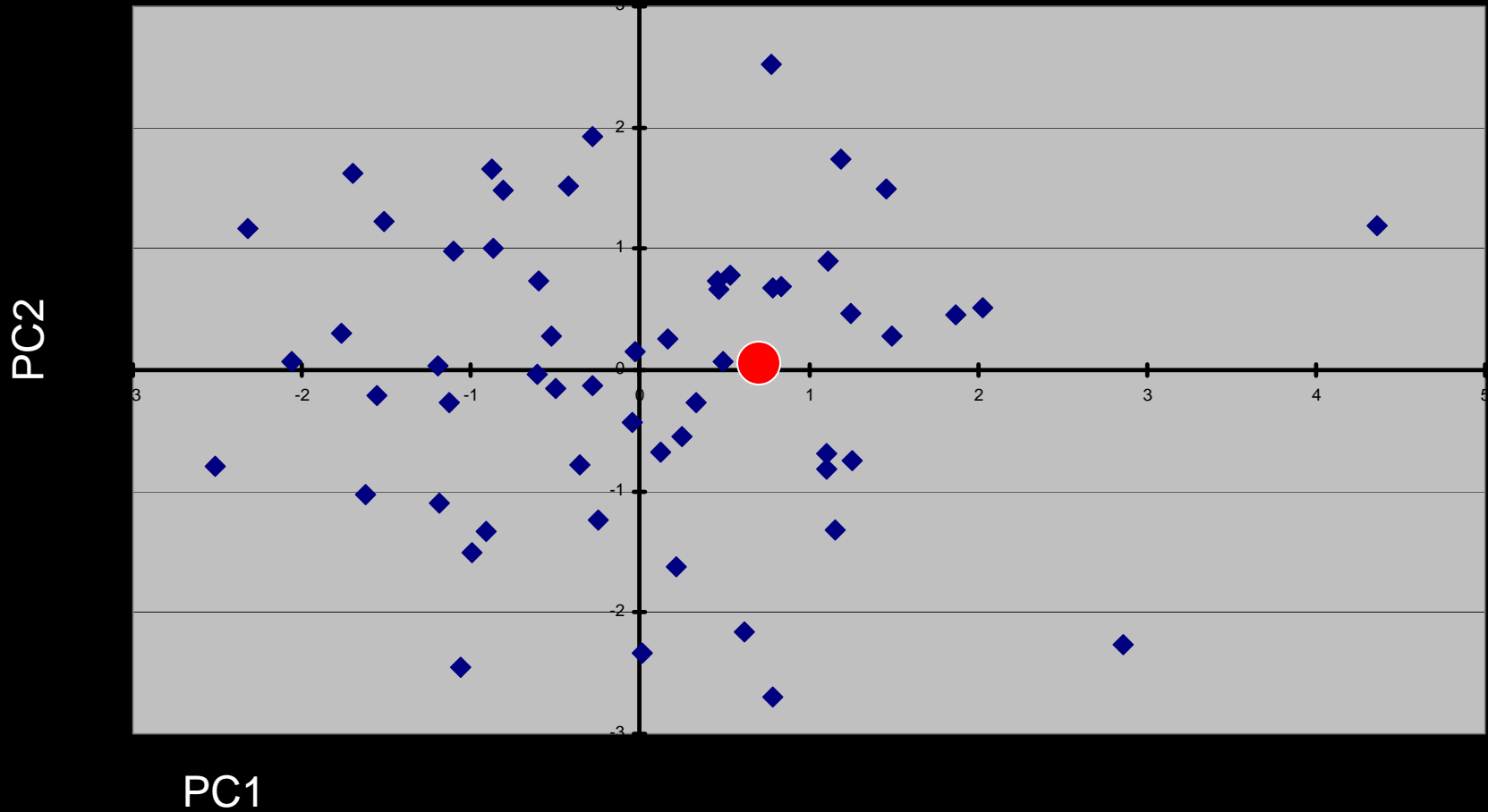
- Reconstruction period: 1400 – 1905, only tree-ring data are available.
- Overlap period: 1906 – 2005, both tree ring data and streamflow data are available.



**Example of projected distribution of tree ring chronologies for overlap years.**

# You Are Here

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```

Distance_ReconPcaEndYear_1997.out - WinVi
File Edit Search Options Windows Help
1400 0 919773778359 1 03618702121 1.16073114553 1.25513813805 1.28501013762 1.29817556085 1.59929923337 1.772
1401 0 646561038747 0 833757311747 0.842376033593 0.884496913167 1.04108467425 1.04349205237 1.05498246415 1.085
1402 0 827183457304 0 992816226473 1.25178850894 1.35066518797 1.48753812434 1.49710862994 1.50239408228 1.530
1403 0 70064370005 0 908472072237 0.905742072237 0.985742072237 1.01254161357 1.03940949663 1.06997770644 1.103
1404 0 633161969892 0 743974693955 0.817935561608 0.867693017899 0.905131281505 0.928855249265 0.998487524969 1.001
1405 0 674998624548 0 917063312363 1.03502014493 1.24971682736 1.25620401784 1.30262257634 1.31585675489 1.344
1406 0 863926830433 0 880927628175 0.89477407439 0.942071241465 1.00988478438 1.07405946428 1.0798619273 1.091
1407 0 452092986015 0 656141926618 0.813473290737 0.861982421224 0.926984163025 0.95120999955 1.01035213716 1.019
1408 0 608462945518 0 803363662409 0.975781892007 0.985781892007 0.99452633861 0.94420435713 0.95946527069 0.940
1409 0 886324347273 0 897378589733 0.97853581652 1.03324734851 1.1191649455 1.14432766535 1.15475023637 1.160
1410 0 702239673198 0 71514597298 0.8600440953 0.92122213696 1.0696601408 1.12404778351 1.19182881265 1.204
1411 0 801876091234 0 833688912985 0.872413648746 0.883573974889 0.954393936449 0.9621714300 0.974610008917 1.000
1412 1 00041438185 1 1159391688 1.13839486142 1.17394553287 1.25581894074 1.28419665511 1.30741025809 1.338
1413 0 775052514318 0 813603848469 0.86313273641 1.07763308337 1.08707758147 1.09537479095 1.33481910119 1.377
1414 0 923316183972 0 942301431104 0.956150125257 1.08286222644 1.24088524681 1.27913208541 1.62491810523 1.764
1415 0 805102518611 0 819172261553 0.82009595533 0.858157362471 1.01781886153 1.02409243073 1.03422487509 1.110
1416 0 701791989613 0 785122611986 0.918874032879 1.01829022302 1.02369452976 1.08217275643 1.10442479715 1.119
1417 1 00017857356 1 04724412804 1.05417097408 1.14293966441 1.15254126427 1.18991243147 1.069740382 1.303
1418 1 04546145762 1 05120952856 1.06180034944 1.09394546147 1.13189827007 1.14348781109 1.1906809684 1.247
1419 0 61576194981 0 713646702915 0.75219797136 0.782426809647 0.819113619171 0.94765910434 0.96343521239 1.040
1420 0 676089489084 0 724778682208 0.866112066101 0.897264926783 0.943311397277 1.00502945792 1.01820587771 1.041
1421 0 999403382623 1 06423154144 1.08000659374 1.23350454355 1.28853441897 1.44537702026 1.45945731002 1.488
1422 0 676547130697 0 87567910578 0.881001205608 1.05158939447 1.05158939447 1.15263996686 1.164410177195 1.16425201981 1.191
1423 0 613457320982 0 93545090224 1.08254070692 1.21573773696 1.2641601062 1.28572789289 1.28667578707 1.409
1424 0 696128537705 1 01153240215 1.03688551443 1.05683212461 1.08661677905 1.09772872876 1.12350145362 1.204
1425 0 885651038656 1 00012556159 1.13563640651 1.17089933666 1.19195147174 1.214460992932 1.24289506878 1.273
1426 0 863202014084 0 987717984725 1.03923686178 1.04324186249 1.04914652342 1.11448479018 1.13245272362 1.152
1427 0 508522388973 0 704923392542 0.776380246803 0.81180027811 0.825195103007 0.85340254621 0.88642038168 0.942
1428 0 563056418275 0 635959082575 0.710225740419 0.752007895688 0.770930602739 0.923672216835 1.10156671916 1.155
1429 0 764522776314 0 779855779571 0.844137864397 0.974091220825 1.02419477995 1.0561216484 1.13997360184 1.185
1430 0 554786269587 0 64468101214 0.709548602265 0.718601664941 0.756073398987 0.757768275424 0.769208487414 0.853
1431 0 889728643952 0 927742793604 0.96846396915 1.07826992024 1.08631648103 1.15668183305 1.1652415197 1.188
1432 0 76035499528 0 78500288633 0.8133990122 0.86775389583 0.87317727471 0.88750287455 0.895766816533 0.938
1433 0 653674873922 0 723201301091 0.877598200659 0.889616032251 0.915680909449 1.01661149754 1.0206005242 1.052
1434 1 27832815104 1 31096994368 1.32079174271 1.52059736952 1.53979298389 1.56533572818 1.56932190565 1.592
1435 1 23981456739 1 289957663623 1.31530499165 1.35611229977 1.43322399136 1.45409016881 1.45488890934 1.584
1436 0 67182513034 0 74858246819 0.748956496915 0.919836166855 0.966268764063 0.98505069258 0.986515271521 1.021
1437 0 602259395134 0 642020950031 1.02827785879 1.10871430018 1.15777706435 1.25186110378 1.2965359905 1.385
1438 1 15296020133 1 25623787524 1.32697543617 1.35303369384 1.35826927814 1.41673406262 1.4940607163 1.532
1439 0 877634543726 0 998618009777 1.05274203473 1.10160398612 1.22664003688 1.26459211686 1.34129710602 1.357
1440 0 634488037246 0 672355134667 0.69239930198 0.692760615247 0.738131071758 0.812883056696 0.872636188893 0.942
1441 0 8972400808 0 900391177757 0.93940748995 0.9523329808 0.98970258375 1.005554859 1.01639195483 1.030
1442 1 00010518377 1 10270634197 1.35714705051 1.54228407922 1.72332474319 1.76912101733 1.79850372092 1.909
1443 0 578804817594 1 04109138293 1.05593407044 1.06356930632 1.1000482019 1.14893364917 1.17904027096 1.207
1444 0 704044211976 1 03016935208 1.06525968008 1.06654534824 1.1312627558 1.16900982812 1.22914865971 1.423
1445 0 963836136268 0 99443751372 1.04729256405 1.14088411243 1.19037689263 1.20227413303 1.4113601596 1.485
1446 0 668949190188 0 991949654489 1.07704013273 1.136826619 1.16282499639 1.1683777793 1.2293231511 1.269
1447 0 556797901779 0 871907668074 0.926511508008 0.92746731817 0.945557414074 0.993416003193 1.01022787193 1.018
1448 0 667991755687 0 697618927202 0.757994950788 0.865199300667 0.956928323877 1.00898587232 1.04145057352 1.042

```



# Finding Similar Years

PC2

14xx

19xx

PC1

ReconPcaEndYear\_1997.out - WinVi

File Edit Search Options Windows Help

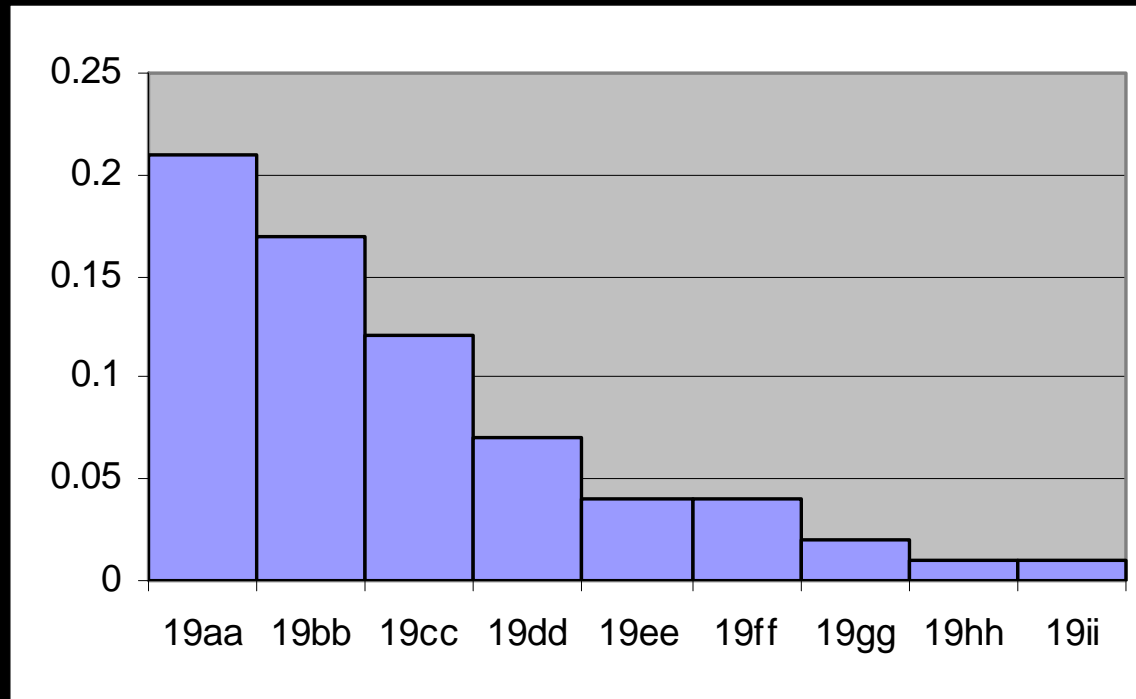


1400	1961	1963	1956	1936	1954	1964	1946	1990	1934	1959
1401	1960	1971	1935	1972	1989	1939	1925	1950	1933	1940
1402	1951	1953	1946	1990	1963	1981	1972	1989	1925	1994
1403	1972	1925	1989	1935	1953	1960	1951	1981	1940	1939
1404	1973	1997	1949	1975	1995	1941	1912	1911	1916	1942
1405	1978	1980	1992	1945	1919	1979	1937	1958	1991	1933
1406	1952	1938	1969	1965	1910	1913	1915	1947	1932	1988
1407	1919	1992	1933	1978	1958	1943	1991	1948	1960	1945
1408	1980	1978	1923	1962	1957	1918	1982	1913	1937	1922
1409	1942	1912	1995	1907	1911	1997	1973	1916	1949	1975
1410	1958	1919	1933	1948	1943	1991	1992	1966	1970	1937
1411	1943	1991	1978	1933	1970	1948	1958	1919	1937	1980
1412	1928	1965	1926	1975	1997	1947	1949	1973	1995	1912
1413	1963	1964	1990	1955	1953	1946	1951	1994	1989	1968
1414	1993	1994	1993	1985	1984	1989	1943	1993	1993	1986

## Year Weighting

---

- Based on similarity distances
- Weight observed years according to similarity



# Weighting Functions

**(1) Bi-square Weight (BSW)**  
Function of distance

$$w_i = \frac{\left[1 - \left(\frac{d(i)}{d(K)}\right)^2\right]^2}{\sum_{i=1}^K \left[1 - \left(\frac{d(i)}{d(K)}\right)^2\right]^2}$$

**(2) Inverse Distance Square Weight (IDW)**  
Function of distance

$$w_i = \frac{1/d_i^2}{\sum_{i=1}^{nt} 1/d_i^2}$$

**(3) One Over K Weight (OKW)**  
Function of number of nearest neighbors K

$$p_j = \frac{1/j}{\sum_{i=1}^K 1/i}$$

In this study  $nt = K = 10$ . The weights ( $w_i$  or  $p_j$ ) are the weights assigned to each neighbor.

## Chronology and Weighting Schemes

---

- Used standardized chronology (STD), 51 chronologies.
- Considered chronologies in the overlap period ending in 1997, 2002, and 2005.

<i>1400 – 1997</i>	<i>32</i>
<i>1400 – 2002</i>	<i>22</i>
<i>1400 – 2005</i>	<i>7</i>

- Three different weighting schemes, BSW (bi-square weight), IDW (inverse distance weight), and OKW (one over K weight).
- Flow reconstruction from 1400-1905.

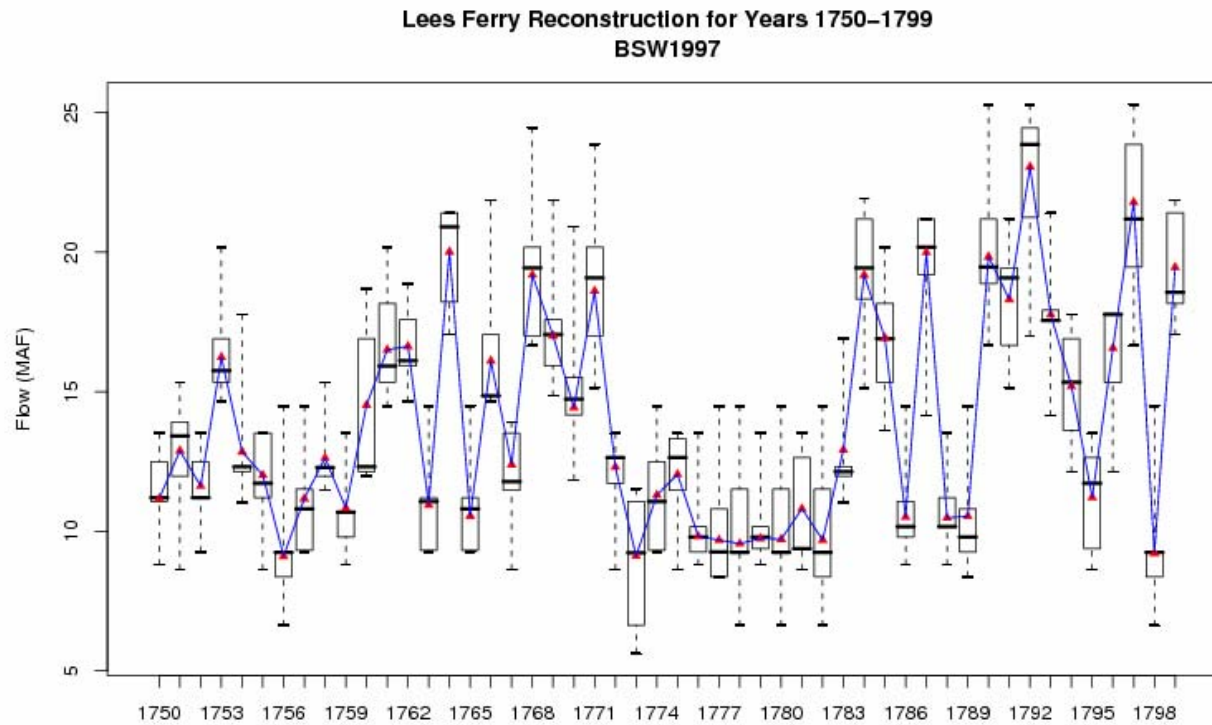
## Temporal Distribution of Chronologies

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<i>Year</i>	<i>Number of Chronologies</i>
1400 - 1404	32
1405 - 1436	34
1437 - 1439	35
1440 - 1449	36
1450 - 1453	37
1454 - 1479	38
1480 - 1507	39
1508 - 1510	40
1511 - 1519	41
1520 - 1523	42
1524 - 1535	43
1536 - 1565	44
1566 - 1568	46
1569 - 1570	47
1571 - 1574	48
1575 - 1583	49
1584 - 1603	50
1604 - 1905	51

- In this case the chronologies in the overlap period end in 1997.

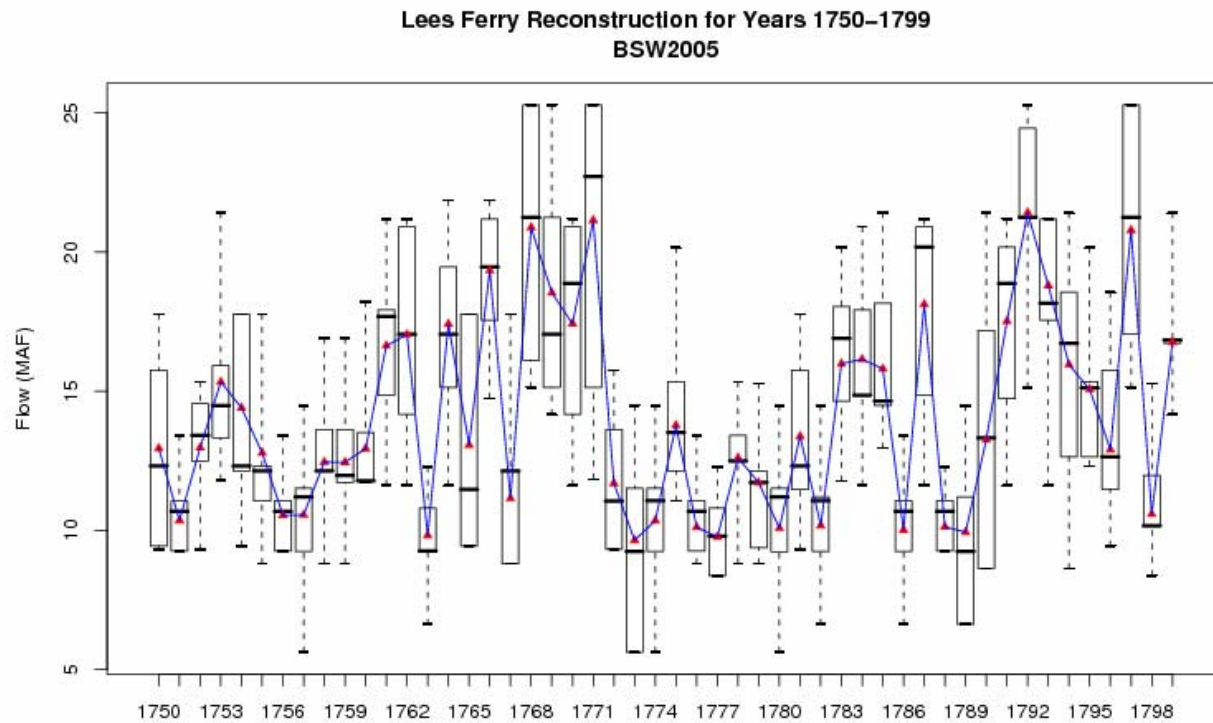
# Streamflow Reconstructions – BSW1997



▲ Expected flow



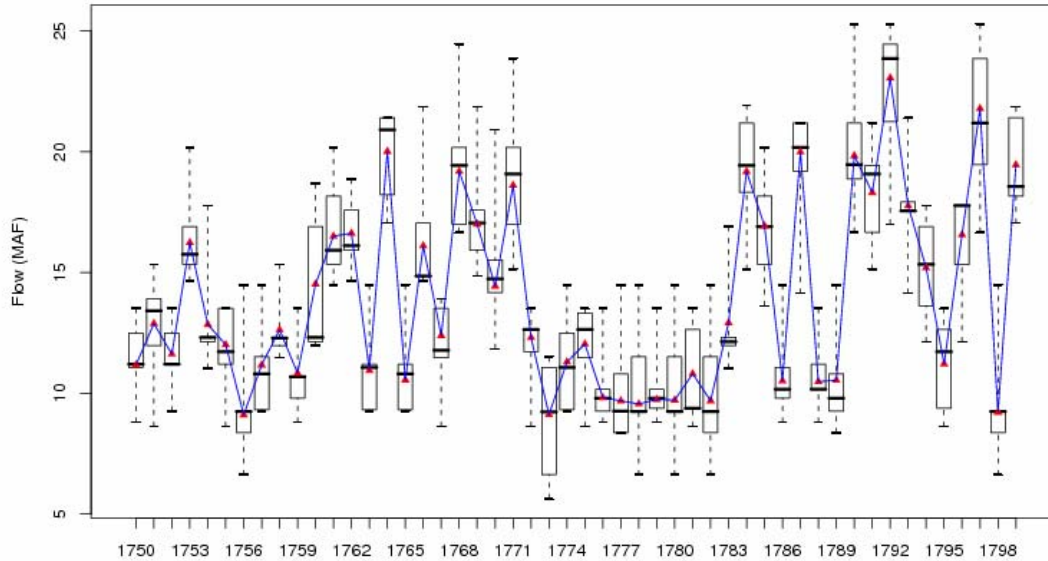
# Streamflow Reconstructions – BSW2005



▲ Expected flow

## BSW 1997

Lees Ferry Reconstruction for Years 1750–1799  
BSW1997



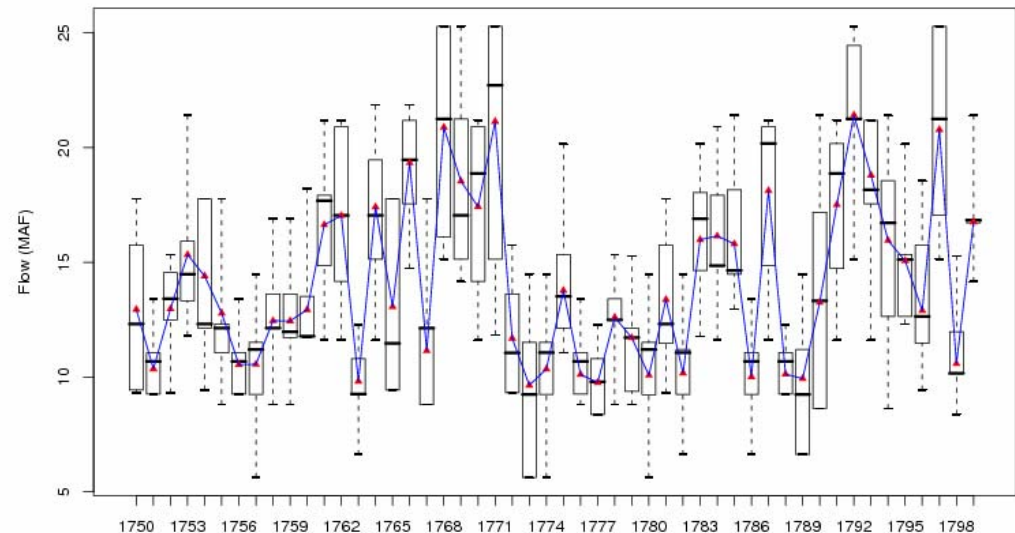
## Sample Size Uncertainty

Less number of Chronologies

- 7 chronologies
- BSW 2005

## BSW 2005

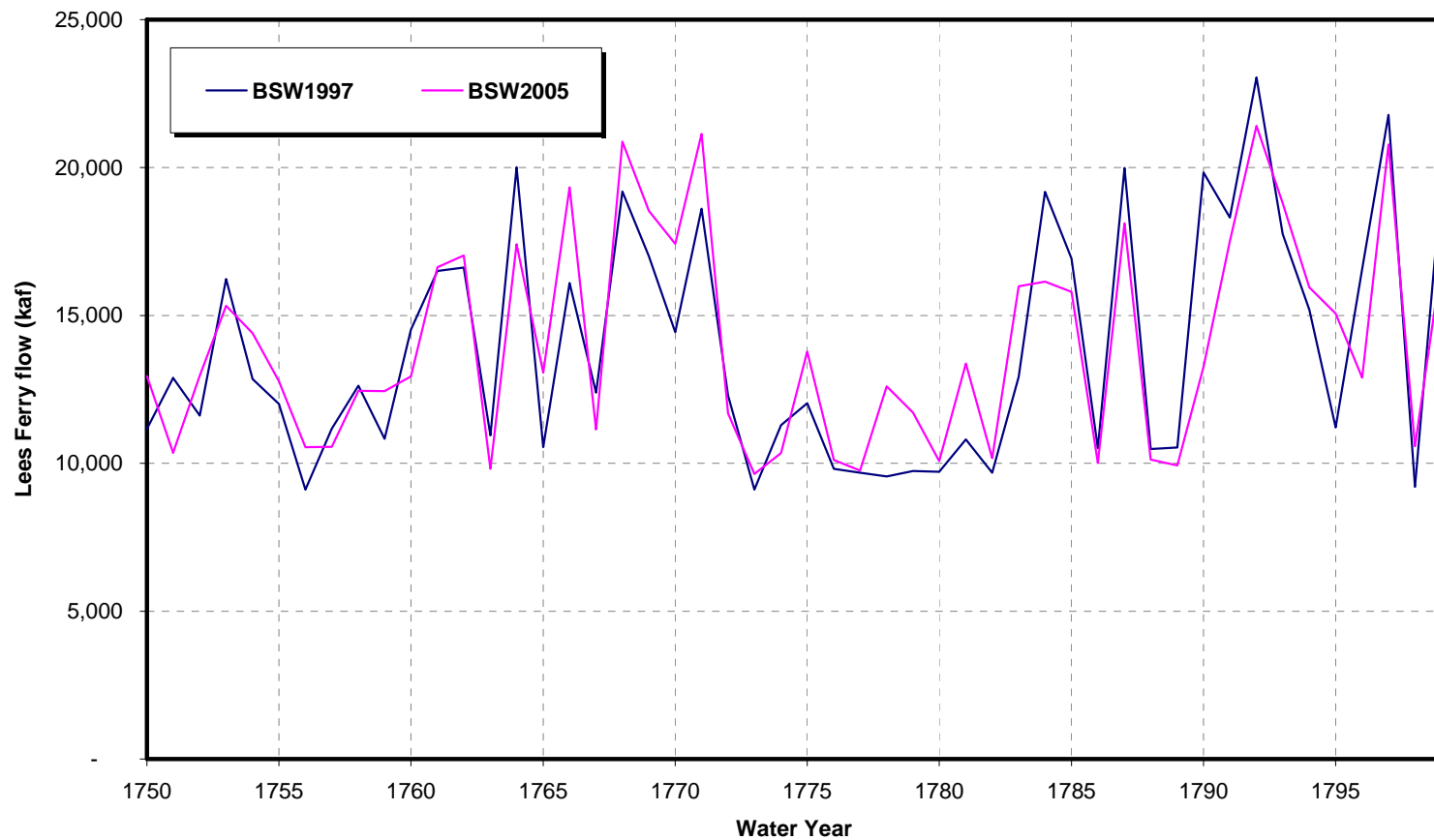
Lees Ferry Reconstruction for Years 1750–1799  
BSW2005



Greater number of  
Chronologies

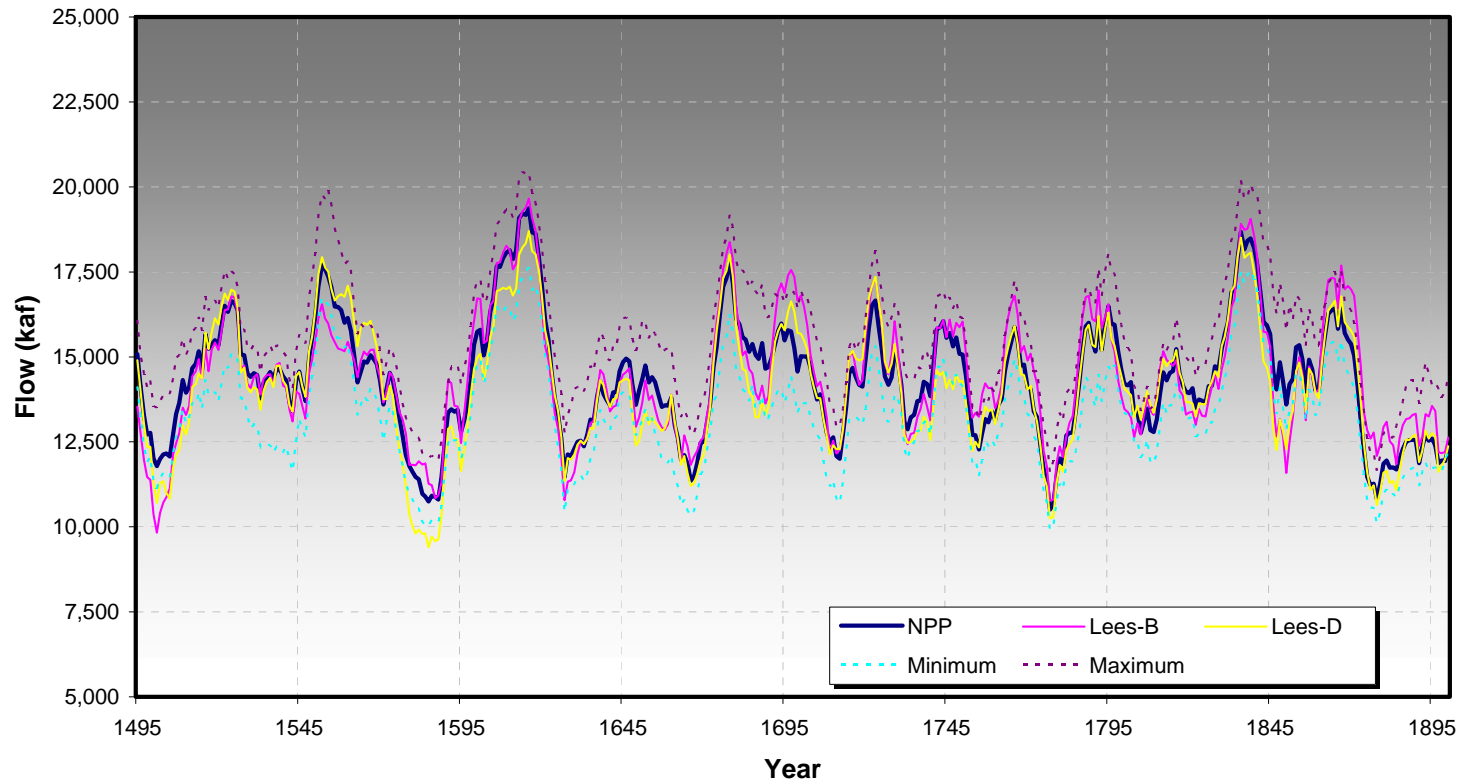
- 51 chronologies
- BSW 1997

# Comparison of Expected Values – BSW 1997 & 2005

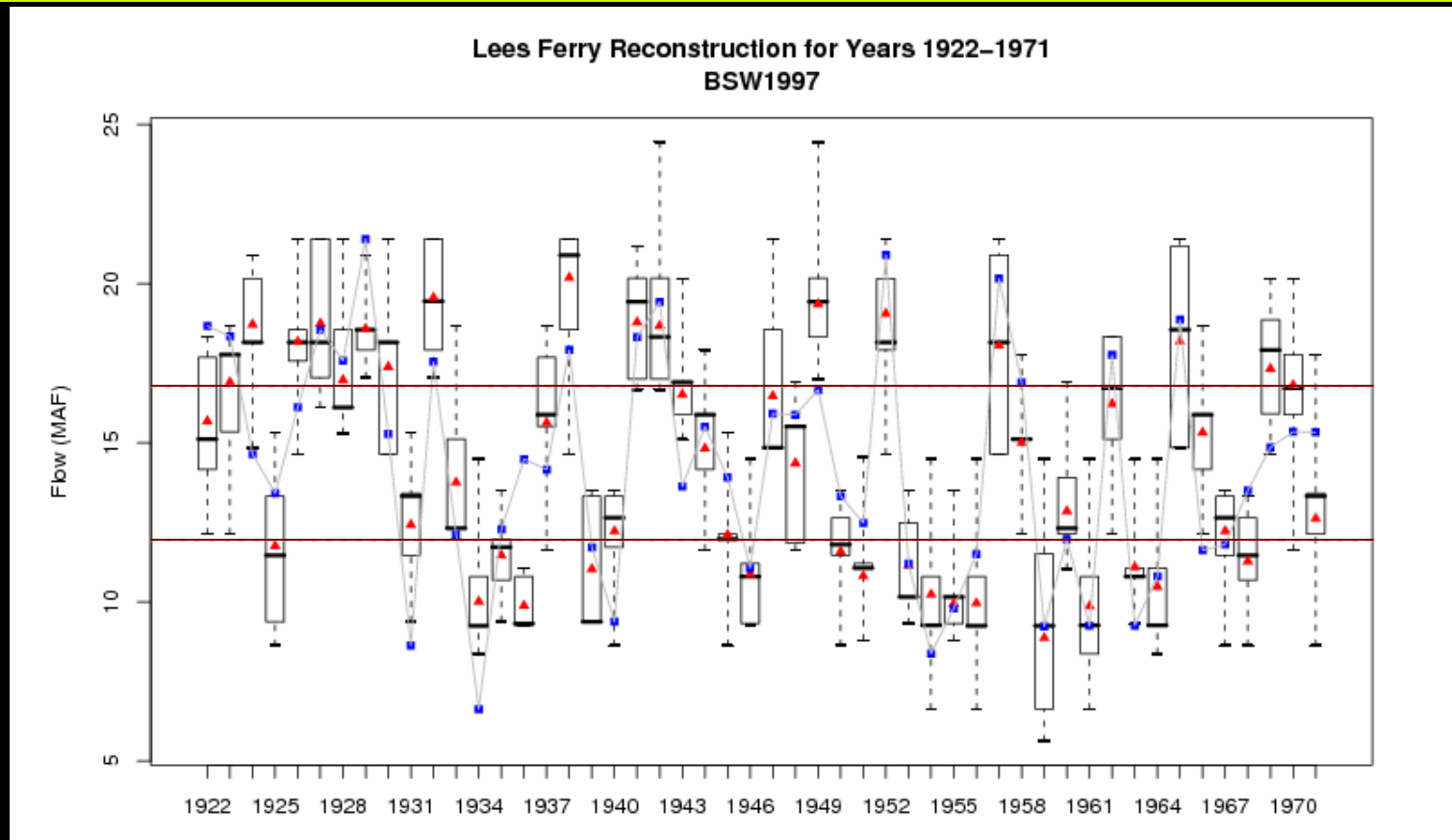


# Comparison of NP With Existing Reconstructions

## Comparison of Paleo Reconstructions Smoothed Using a 11-year Moving Window



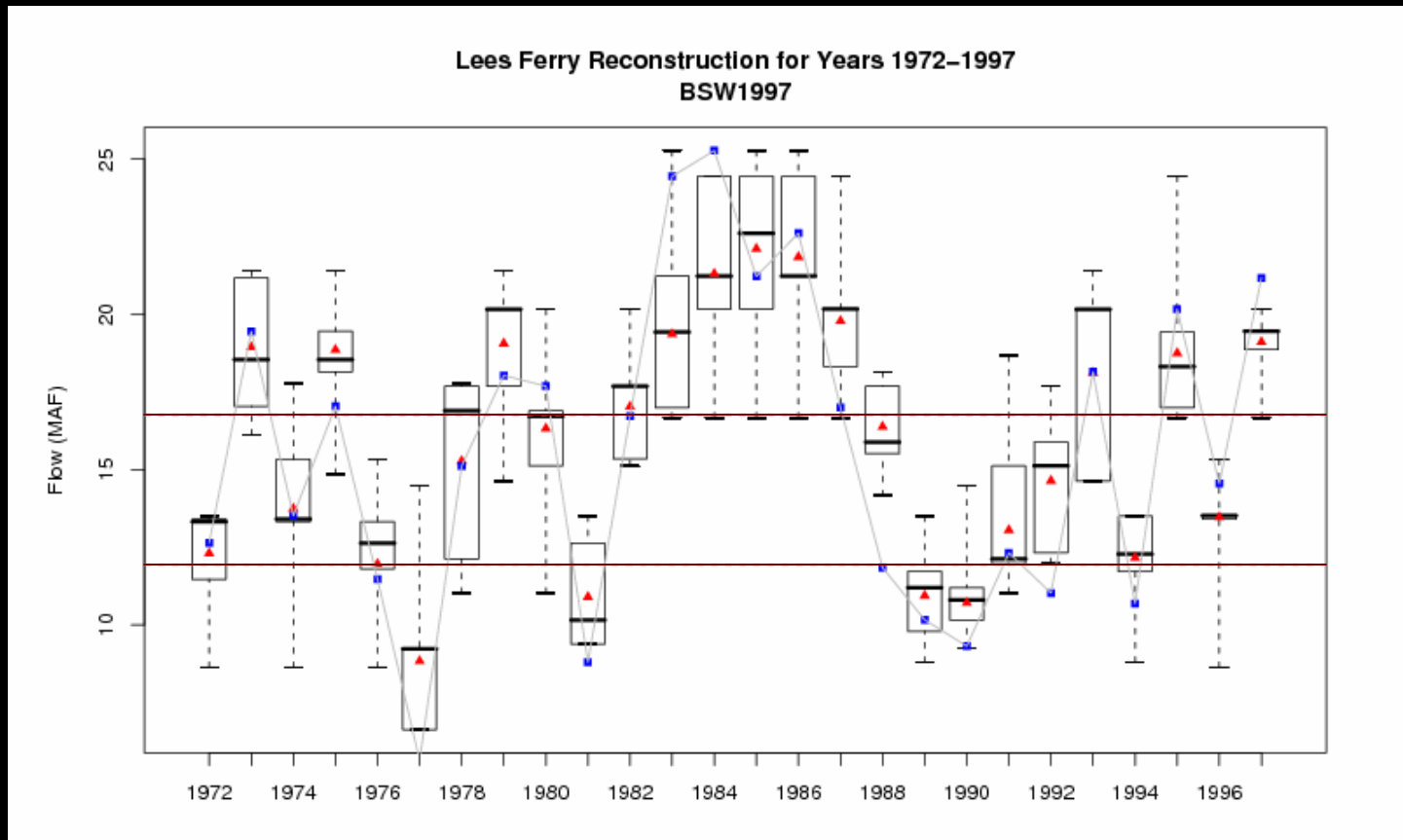
# Overlap Period Verification – 1922 to 1971



Red triangle – Expected flow from NP reconstructions

Blue square – Observed/calculated natural flow

# Overlap Period Verification – 1972 to 1997



Red triangle – Expected flow from NP reconstructions

Blue square – Observed/calculated natural flow

# Overlap Period Verification, 1922 – 1997 Weighting Scheme BSW

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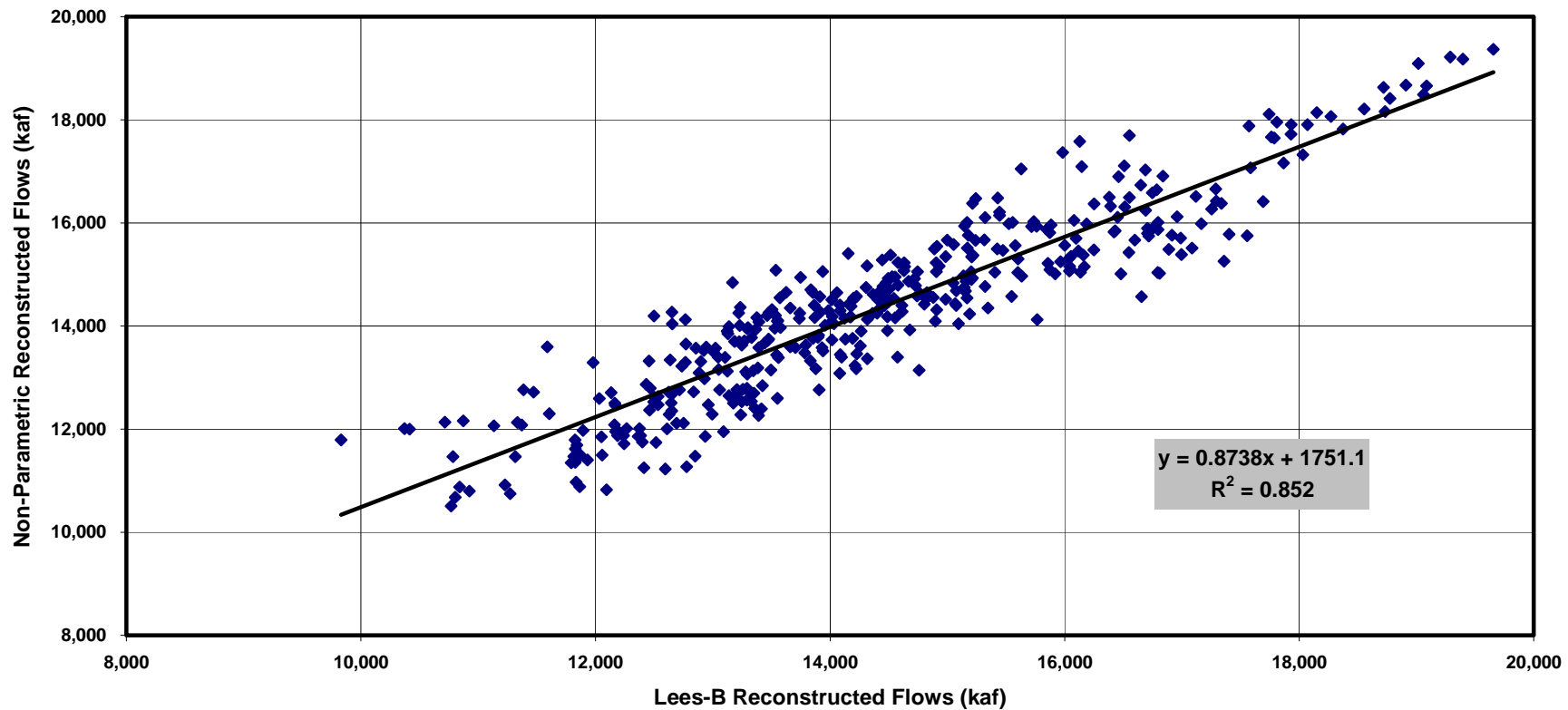
## Joint Probability

	Odry	Oavg	Owet
Sdry	0.1974	0.0789	0.0000
Savg	0.1053	0.1842	0.0526
Swet	0.0000	0.0921	0.2895

**Hit Rate** 0.6711

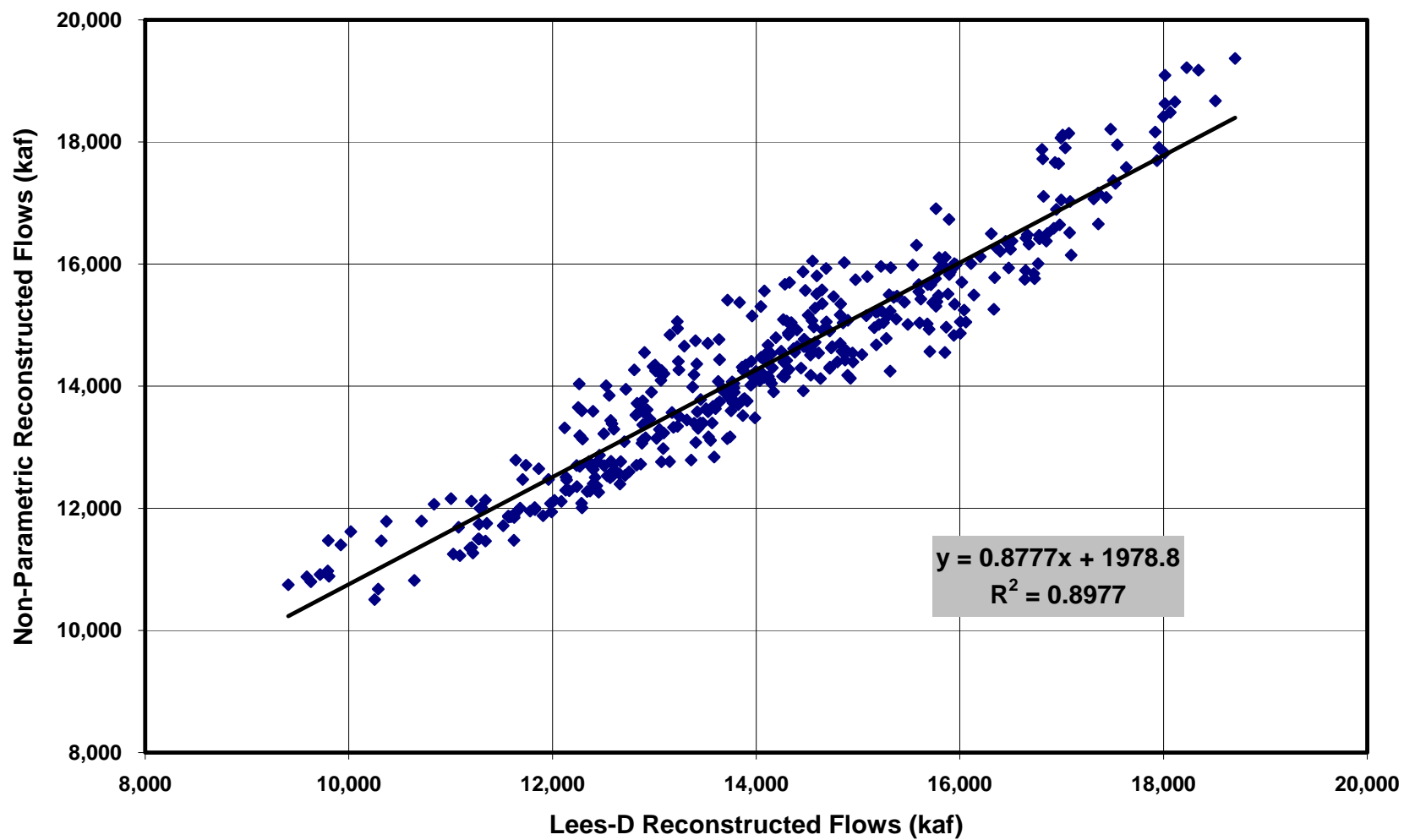
		Observed	
		Dry	Wet
Simulated	Dry	33 (0.4342)	2 (0.0263)
	Wet	5 (0.0658)	36 (0.4737)

Lees-B and NPP Scatter Plot

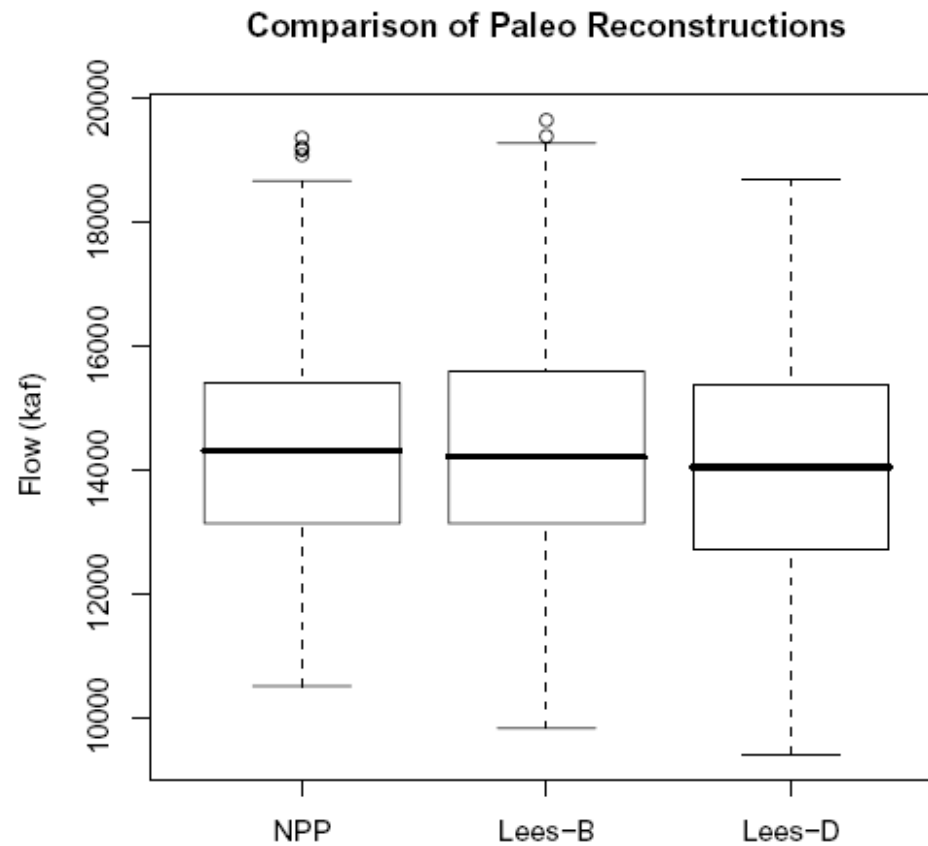




## Lees-D and NPP Scatter Plot



# Comparison



## Conclusions - Advantages of NP Reconstruction

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- NP techniques do not assume:
  - Normality
  - Linearity
- Thus avoid artifacts from assumptions inherent in linear regression.
- Insensitive to modeled data--Can model structured data (e.g. model outputs) directly.
- Single parameter model – number of neighbors.
- Data driven and does not require model fitting or calibration.
- Unsymmetrical error bounds.

## Acknowledgements

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- Jeff Lukas, INSTAAR, CIRES/University of Colorado
- Balaji Rajagopalan, University of Colorado
- Connie Woodhouse, University of Arizona
- Jim Prairie, USBR
- Brad Udall, CIRES/NOAA
- Terry Fulp, USBR

**Thank You!**